

# Functional Programming for Trade Management and Valuation

Seminar on Functional High Performance Computing in Finance  
December 14, 2010

Martin Elsman  
SimCorp A/S

# The Financial Contracts Market

Banks (and other financial institutions) use financial contracts for both

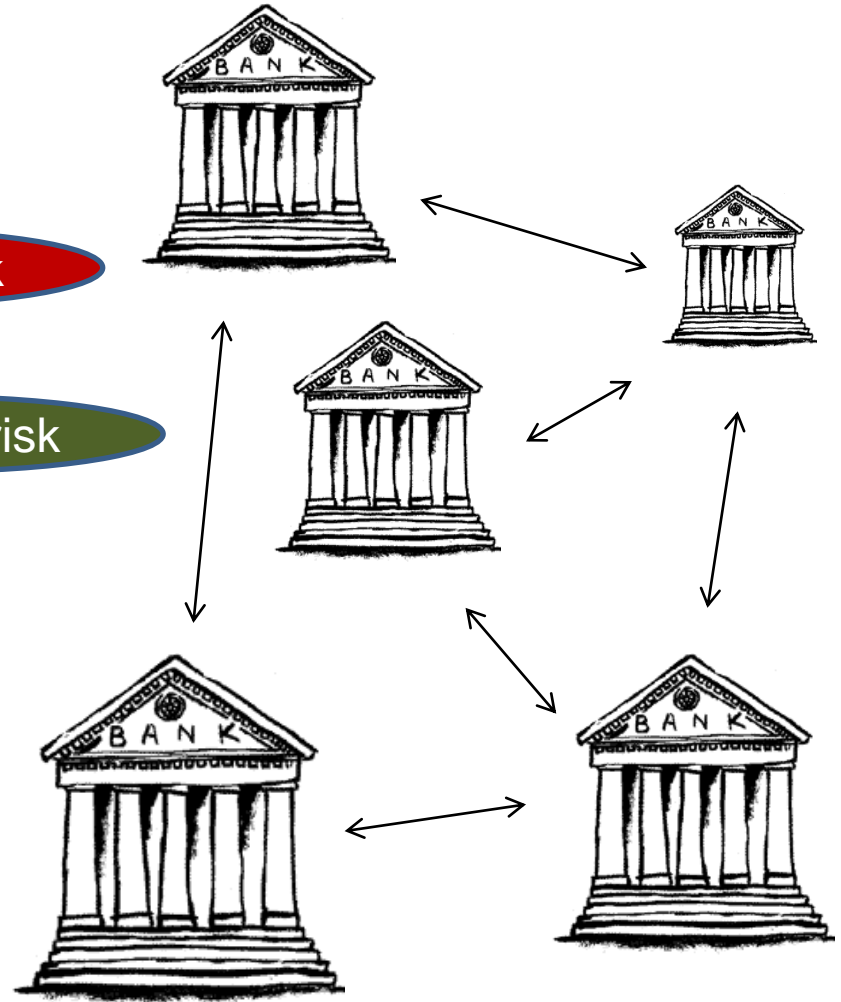
- Speculation

Increase risk

- Insurance (hedging)

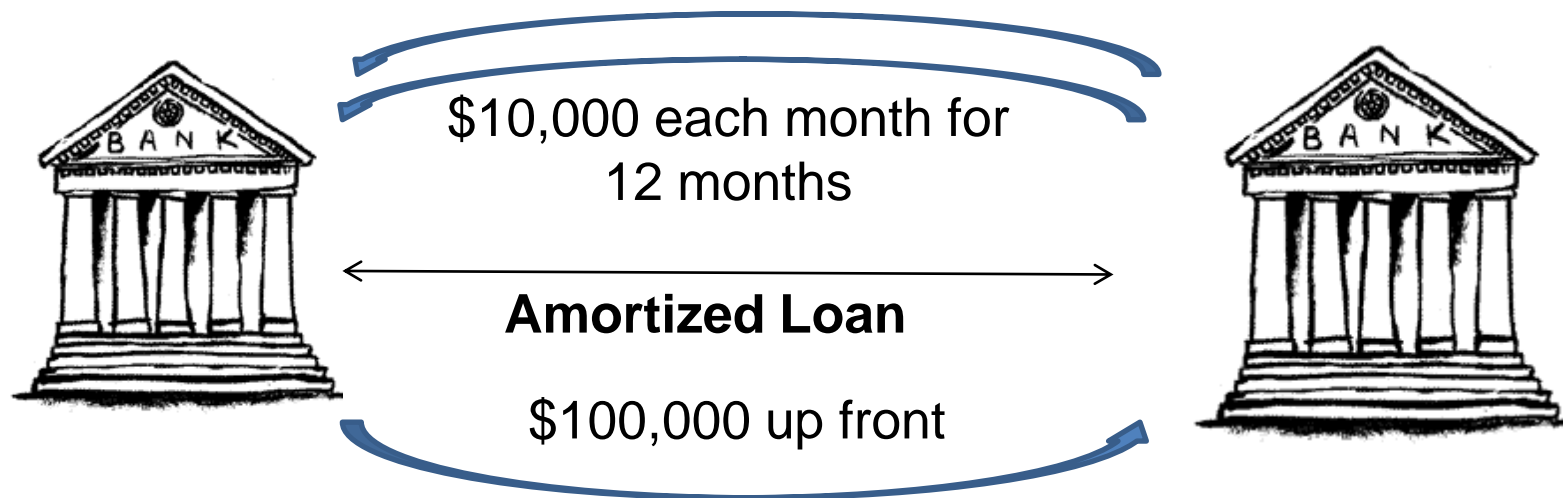
Decrease risk

Many contracts are "**Over The Counter**" (OTC) contracts, which are negotiated agreements between a bank and another bank (its counter party).



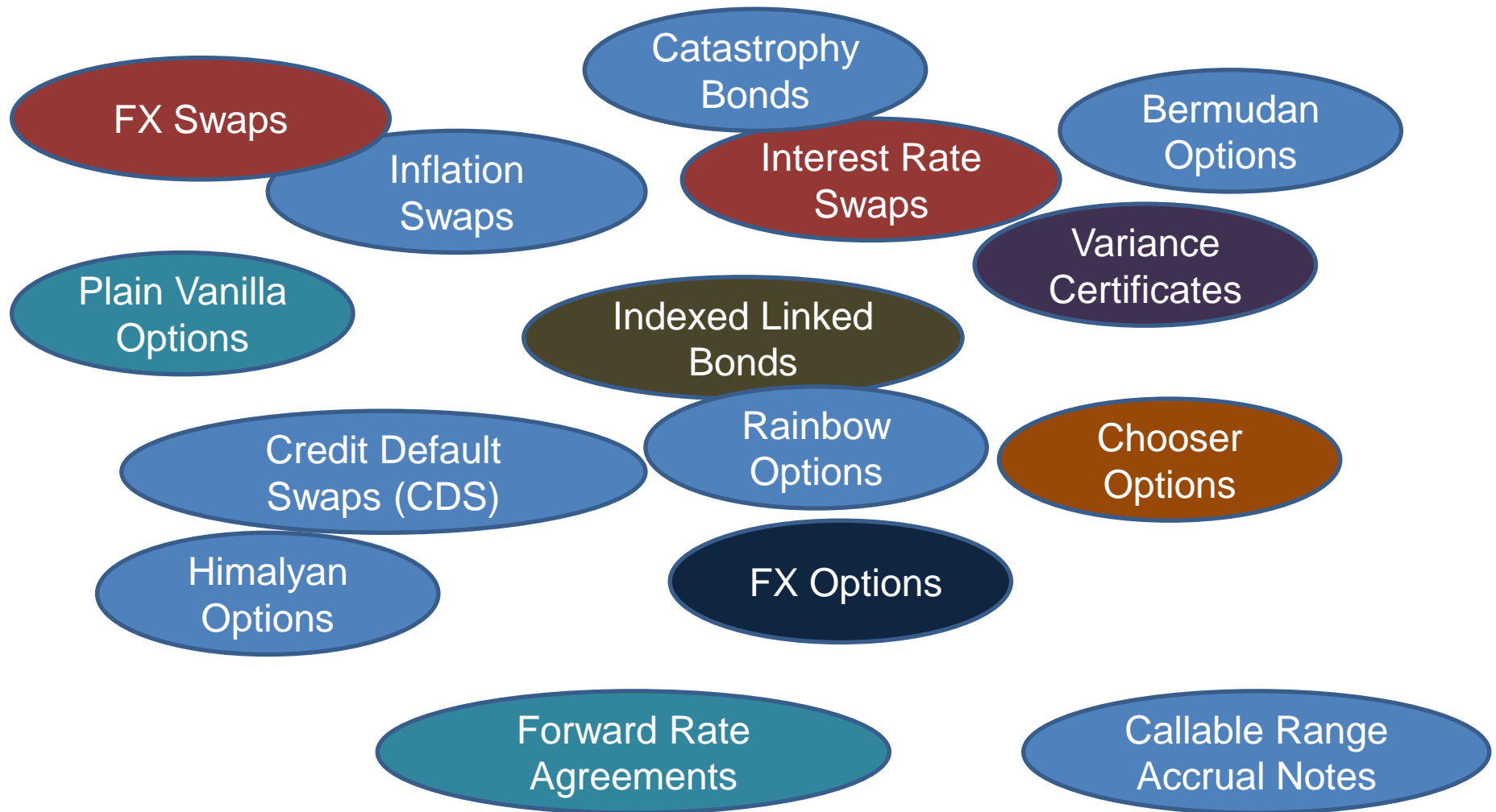
# The Term Sheet – the financial contract

- A financial contract is typically agreed upon on a so-called "Term Sheet".
- The *term sheet* specifies the financial flows (amounts, dates, etc.) and under which *conditions* a flow should happen.
- Flows can go in both directions.



- A **derivative** is a contract that depends on an underlying entity (e.g., a stock)

# Many Types of Financial Contracts are Traded



# How do the Banks Keep Track?

- **Many Problems:**

- Financial contracts need management
  - *fixings, decisions, corporate actions, ...*
- Banks must report daily on their total value of assets
- Banks must control risk (counterparty risk, currency risk, ...)
- Banks need to know about future cash flows

- **A Solution:**

- **Specify financial contracts in a domain specific language!**
- **Use a functional programming language (e.g., ML)**



Algebraic properties



Simple reasoning

# The SimCorp XpressInstruments Solution

Instrument specific input

Instruments are specified in an *instrument modeling language*

Once loaded, a **portfolio manager** may **instantiate** an instrument to create contracts.

The **instrument knows** what input to ask for.

**Wall-to-wall** (Front Office, Middle Office, Back Office) contract management

0 - Main window for XpressInstruments

Home Contract XpressInstrument Editor Help

Financial Instruments

- Autocallable Down and In
- Autocallable Down and In Index
- Autocallable Index
- Autocallable Worst-of
- Basket Certificate
- Bonus Certificate
- Callable Path Dependent Floater
- Callable Range Accrual Note
- Chooser Option
- Compound Option
- Discount Certificate
- Equity Basket Dispersion
- Everest
- Himalaya
- IR-CCY Swap
- Index Tracker
- Libor Plus
- MinMax VolBond
- Plain Vanilla Option
- Rainbow Option
- Snowball / TARN
- Spread Express
- Target Redemption Note
- Variance Certificate
- Variance Swap
- Variance Swap - Explicit Dates
- Variance Swap/Note
- Zero Coupon Inflation Swap

Security Identification

Security ID  ISIN  State of data

Security/Serial No.  Security name  A/L classification

Security type  Security group  XpressInstrument

Financial Instrument Characteristics

Contract Definition Past Events Future Flows Future Fixings and Decisions Simulate Fixings and Decisions Pricing

Underlying

Maturity date

Underlying call

Fixing date	<input type="text" value="14-10-2010"/>	...
Maturity date	<input type="text" value="28-10-2010"/>	...
Strike	<input type="text" value="45,00"/>	
Currency	<input type="text" value="DKK"/>	
Automatic	<input type="text" value="True"/>	

Underlying put

Fixing date	<input type="text" value="16-10-2010"/>	...
Maturity date	<input type="text" value="24-10-2010"/>	...
Strike	<input type="text" value="60,00"/>	
Currency	<input type="text" value="EUR"/>	
Automatic	<input type="text" value="True"/>	

Business Classes Business Codes

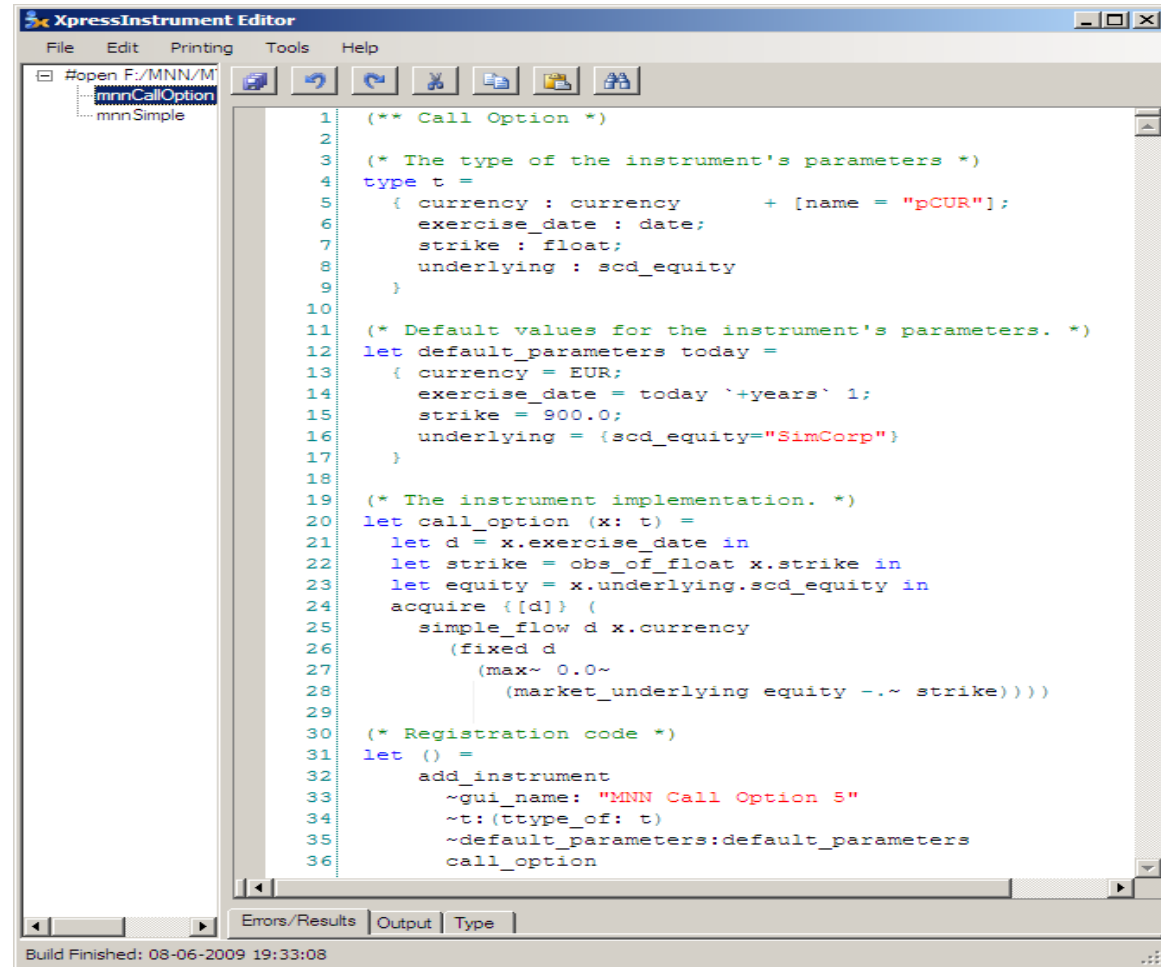
My Layout Update 1/2

LexiFi  
Technology  
Inside!!

# The SimCorp XpressInstruments Solution

Instrument IDE

- Instruments are **written** by SimCorp consultants or by banks themselves:
- Newly written instruments may be **loaded** into the system **instantaneously**
- **Notice: Arbitrary short time-to-market**



```
1  (** Call Option *)
2
3  (* The type of the instrument's parameters *)
4  type t =
5  { currency : currency          + [name = "pCUR"];
6    exercise_date : date;
7    strike : float;
8    underlying : scd_equity
9  }
10
11 (* Default values for the instrument's parameters. *)
12 let default_parameters today =
13 { currency = EUR;
14   exercise_date = today `+years` 1;
15   strike = 900.0;
16   underlying = {scd_equity="SimCorp"}
17 }
18
19 (* The instrument implementation. *)
20 let call_option (x: t) =
21 let d = x.exercise_date in
22 let strike = obs_of_float x.strike in
23 let equity = x.underlying.scd_equity in
24 acquire {[d]} (
25   simple_flow d x.currency
26   (fixed d
27    (max~ 0.0~
28     (market_underlying equity -.~ strike))))
29
30 (* Registration code *)
31 let () =
32 add_instrument
33 ~gui_name: "MNN Call Option 5"
34 ~t:(ttype_of: t)
35 ~default_parameters:default_parameters
36 call_option
```

Build Finished: 08-06-2009 19:33:08

# Constructing Contract Management Software in Standard ML

## Basics:

```
(* Currencies *)
datatype currency = EUR | DKK

(* Observables *)
datatype obs =
  Const of real
  | Underlying of string * Date.date
  | Mul of obs * obs
  | Add of obs * obs
  | Sub of obs * obs
  | Max of obs * obs
```

In reality, there are, of course, more currencies...

**Observable:** algebra over measurable time-changing entities (e.g., Carlsberg stock)



# The Contract Language as a Standard ML Datatype

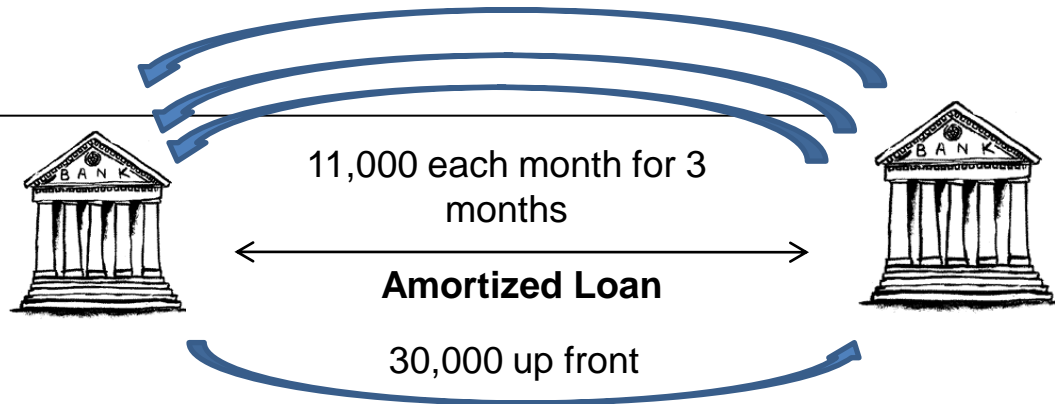
Flow of one unit

Acquire the underlying contract at specific date

```
(* Contracts *)
datatype contract =
  One of currency
  | Scale of obs * contract
  | All of contract list
  | Acquire of Date.date * contract
  | Give of contract

(* Shorthand notation *)
fun flow(d,v,c) = Acquire(d,Scale(Const v,One c))
val zero = All []
```

# Example Financial Contracts



```
(* Simple amortized loan *)
```

```
val ex1 =
```

```
  let val coupon = 11000.0
```

```
      val principal = 30000.0
```

```
  in All [Give(flow(?"2011-01-01", principal, EUR)),
```

```
          flow(?"2011-02-01", coupon, EUR),
```

```
          flow(?"2011-03-01", coupon, EUR),
```

```
          flow(?"2011-04-01", coupon, EUR) ]
```

```
end
```

```
(* Cross currency swap *)
```

```
val ex2 =
```

```
  All [Give(
```

```
    All [flow(?"2011-01-01", 7000.0, DKK),
```

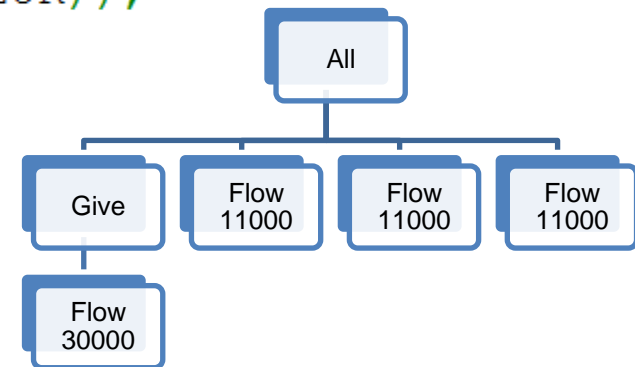
```
         flow(?"2011-02-01", 7000.0, DKK),
```

```
         flow(?"2011-03-01", 7000.0, DKK) ]),
```

```
    flow(?"2011-01-01", 1000.0, EUR),
```

```
    flow(?"2011-02-01", 1000.0, EUR),
```

```
    flow(?"2011-03-01", 1000.0, EUR) ]
```



**Notice:** flows in different currencies

## A Somewhat more Complex Example

```
(* Call option on "Carlsberg" stock *)
val equity = "Carlsberg"
val maturity = ?"2012-01-01"
val ex4 =
  let val strike = 50.0
      val nominal = 1000.0
      val obs =
        Max(Const 0.0,
             Sub(Underlying(equity, maturity),
                 Const strike))
      in Scale(Const nominal,
               Acquire(maturity, Scale(obs, One EUR)))
    end
```

**Meaning:** Acquire at maturity the amount (in EUR), calculated as follows ( $P$  is price of Carlsberg stock at maturity):

$$\textit{nominal} * \max(0, P - \textit{strike})$$

# Now What?

---

- We have now defined some contracts, but **what can we do with the definitions?**
  - Report on the **expected future cash flows**
  - Perform **management operations**:
    - Advancement (simplify contract when time evolves)
    - Corporate action (stock splits, merges, catastrophic events, ...)
    - Perform fixing (simplify contract when an underlying becomes known)
  - Report on the **value (price) of a contract**
  - ...

# Expected Future Cash Flows

```
(* Future cash flows *)  
fun noObs _ = raise Fail "noObs"  
val _ = println "\nEx1 - Cash flows for simple amortized loan:"  
val _ = println (cashflows noObs ex1)
```

Ex1 - Cash flows for simple amortized loan:

2011-01-01	Certain	EUR	~30000.0000000
2011-02-01	Certain	EUR	11000.0000000
2011-03-01	Certain	EUR	11000.0000000
2011-04-01	Certain	EUR	11000.0000000

```
val _ = println "\nEx2 - Cash flows for cross-currency swap:"  
val _ = println (cashflows noObs ex2)
```

Ex2 - Cash flows for cross-currency swap:

2011-01-01	Certain	EUR	1000.00000000
2011-01-01	Certain	DKK	~7000.00000000
2011-02-01	Certain	EUR	1000.00000000
2011-02-01	Certain	DKK	~7000.00000000
2011-03-01	Certain	EUR	1000.00000000
2011-03-01	Certain	DKK	~7000.00000000

When a contract is *given away*, flows are inverted

It is possible to define a function **cashflows** that collects information about the future cash flows of a contract.

# Contract Management and Contract Simplification

Fixing also advances contract

```
(* Stock option cash flows assuming underlying stock price of 79.0 *)
val _ = println "\nEx4 - Cash flows on stock option (Strike:50,Price:79):"
val _ = println (cashflows (fn _ => Const 79.0) ex4)

(* Contract management *)
val ex5 = fixing(equity,maturity,83.0) ex4
val _ = println "\nEx5 - Call option with fixing 83"
val _ = println ("ex5 = " ^ pp ex5)
val ex6 = fixing(equity,maturity,46.0) ex4
val _ = println "\nEx6 - Call option with fixing 46"
val _ = println ("ex6 = " ^ pp ex6)
```

Observable underlyings may introduce uncertainties

## Output:

```
Ex4 - Cash flows on stock option (Strike:50,Price:79):
2012-01-01 Uncertain EUR 29000.0000000
```

```
Ex5 - Call option with fixing 83
ex5 = Scale(33000.0000000,One(EUR))
```

```
Ex6 - Call option with fixing 46
ex6 = zero
```

Contracts are simplified due to calls to the fixing function

# Valuation (pricing)

```
(* Valuation (Pricing) *)
structure FlatRate = struct
  fun discount d0 d amount rate =
    let val time = real(Date.diff d d0) / 360.0
    in amount * Math.exp(~ rate * time)
    end
  fun price d0 (R : currency -> real)
    (FX: currency * real -> real) t =
    let val flows = cashflows0 noE t
    in List.foldl (fn ((d,cur,v,_),acc) =>
      acc + FX(cur,discount d0 d v (R cur)))
      0.0 flows
    end
end

end

fun FX(EUR,v) = 7.0 * v
  | FX(DKK,v) = v
fun R EUR = 0.04
  | R DKK = 0.05

val p1 = FlatRate.price (? "2011-01-01") R FX ex1
val p2 = FlatRate.price (? "2011-01-01") R FX ex2
val _ = println("\nPrice(ex1) : DKK " ^ Real.toString p1)
val _ = println("\nPrice(ex2) : DKK " ^ Real.toString p2)
```

**Notice:** This model is a bit too simple – we assume the FX-rate is constant...

## Output:

```
Price(ex1) : DKK 19465.9718165
Price(ex2) : DKK 17.3909947790
```

# What is Missing?

---

- Proper **date handling** (holidays, business conventions; Act/30, Act/Act, ...)
- **Easy GUI** specification
- More **combinators** (e.g., american optionality, dynamic dates, ...)
- More **functionality** (e.g., accrual interest)
- Support for **corporate actions** and **catastrophic events**
- **Well-formedness** of contracts... *Disallow acquire of flow in the past*
- Proper **stochastic models** and underlying machinery (**Sobol sequences** for **monte-carlo simulations**) for pricing and calibration
  - Support for linking with external models (e.g., FINCAD)

**FINCAD**



# Conclusions

---

- **Functional programming**
  - Is **declarative: Focus on what** instead of how
  - Is **value oriented** (functional, persistent data structures)
  - Eases **reasoning** (formal as well as informal)
  - Eases **concurrent processing** (e.g., for improved parallelism)
- **SimCorp** not the only company (or bank) that has recognized the value of functional programming for the financial industry
  - LexiFi (See ICFP'00 paper by Peyton-Jones, Eber, Seward)
    - **Engine is used by SimCorp!**
  - Jane Street Capital (focus on electronic trading)
  - Societe Generale, Credit Suisse, Standard Chartered, ...
  - Contract "Pay-off" specifications are often written in a functional style

**LexiFi**

## Appendix: Observable evaluation function

```
(* Evaluation utility function on observables *)
exception Eval
fun eval E obs =
  let fun max r1 r2 = if r1 > r2 then r1 else r2
      in case obs of
          Const r => r
        | Underlying arg =>
            let val obs = E arg
                in case obs of
                    Underlying arg1 =>
                      if arg = arg1 then raise Eval
                      else eval E obs
                  | _ => eval E obs
                end
          | Mul(obs1,obs2) => eval E obs1 * eval E obs2
          | Add(obs1,obs2) => eval E obs1 + eval E obs2
          | Sub(obs1,obs2) => eval E obs1 - eval E obs2
          | Max(obs1,obs2) => max (eval E obs1) (eval E obs2)
        end
  end
```

# Appendix: Observable Simplification

## – preparing for Contract Management

```
(* Try to simplify an observable expression *)
fun simplify E obs =
  let fun simpl opr o1 o2 =
        opr(simplify E o1, simplify E o2)
      in (Const(eval E obs))
        handle _ =>
          case obs of
            Const _ => obs
          | Underlying _ => obs
          | Mul(o1,o2) => simpl Mul o1 o2
          | Add(o1,o2) => simpl Add o1 o2
          | Sub(o1,o2) => simpl Sub o1 o2
          | Max(o1,o2) => simpl Max o1 o2
        end
end
```

# Appendix: Future Cash Flows

Propagate scale factor to resolve amount

```
(* Future Cash Flows *)
fun cashflows0 E t =
  let fun flows s d c t =
        case t of
          One cur =>
            [(d,cur,s,if c then Certain else Uncertain)]
        | Scale(obs,t) =>
            flows (s * Obs.eval E obs) d
                  (c andalso Obs.certainty obs) t
        | All ts => List.concat (map (flows s d c) ts)
        | Acquire(d,t) => flows s d c t
        | Give(t) => flows (~s) d c t
      in
        val res = flows 1.0 (today()) true t
      in Listsort.sort
         (fn (r1,r2) => Date.compare(#1 r1,#1 r2))
         res
      end
  end

fun cashflows E t : string =
  let fun pp (d,cur,r,c) =
        Date.toString d ^ " " ^ pp_certainty c ^ " " ^
        pp_cur cur ^ " " ^ Real.toString r
      in
        val res = cashflows0 E t
      in String.concatWith "\n" (List.map pp res)
      end
  end
```

Observable underlyings may introduce uncertainties

When a contract is given away, flows are inverted

# Appendix: Contract Simplification

```
(* Contract Management *)
fun simplify d0 E t =
  case t of
    All ts =>
      let val ts = map (simplify d0 E) ts
      in case List.filter (fn All[] => false | _ => true) ts of
          [t] => t
        | ts => All ts
      end
  | Give(All[]) => All[]
  | Scale(obs,All[]) => All[]
  | Give(All ts) => simplify d0 E (All(map Give ts))
  | Scale(obs,All ts) =>
    simplify d0 E (All (map (fn t => Scale(obs,t)) ts))
  | Scale(obs,t) =>
    (case Scale(simplify_obs E obs,simplify d0 E t) of
      Scale(o1,Scale(o2,t)) =>
        simplify d0 E (Scale(Mul(o1,o2),t))
    | Scale(obs,All[]) => All[]
    | t as Scale(Const r,_) =>
      if Real.==(r,0.0) then zero else t
    | t => t)
  | Acquire(d,t) =>
    if Date.diff d0 d >= 0 then simplify d0 E t
    else Acquire(d,simplify d0 E t)
  | Give t =>
    (case Give(simplify d0 E t) of
      Give(Give t) => simplify d0 E t
    | t => t)
  | One _ => t
```

Complete contract simplifier.

**Scale and Give** constructors are propagated downwards and merged.

**Acquire constructors** are resolved, given the argument date to simplify (d0).

The environment (E) is propagated to the observable simplifier.

## Appendix: Contract Management Using "simplify"

```
(* Apply a fixing to a contract *)
fun fixing (name,date,value) t =
  let fun E arg =
        if arg = (name,date) then Obs.Const value
        else Obs.Underlying arg
      in simplify date E t
    end
```

```
(* Remove the past from a contract *)
fun advance d t =
  let val t = simplify d noE t
      fun adv t =
          case t of
            One _ => zero
          | Scale(obs,t) => Scale(obs, adv t)
          | Acquire _ => t
          | Give t => Give(adv t)
          | All ts => All(map adv ts)
        in simplify d noE (adv t)
      end
```